BANKING AND SYSTEMIC CRISIS

LECTURE NOTES – SET #2

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INTRODUCTION

- The approach for regulating banks deployed by the Basel II-III agreements is centered on the idea of imposing capital requirements based on an internal assessment of risks [internal ratings based (IRB) approach]
- Banks assess the risk to which they are exposed by recurring to statistical calculations measuring the Value-at-Risk (VaR) of their assets as prices vary
- The urge to action coming from price movements generates an externality: in an attempt to mitigate its own risk exposure, a bank contributes to spread the risk all over the financial system
- Systemic risk is endogenous: price changes and balance-sheet adjustments reinforce each other to amplify small shocks

INTRODUCTION (CONT'D)

- The interaction between leverage (ratio between total assets and equity) and balance sheet size is the key
- For households, leverage is inversely related to total asset
- Example:
  - In t = 0 you buy a home which costs 100, using 20 of equity and 80 of debt. The leverage is 100/20 = 5
  - In t =1 the price of the house goes to 120. Debt is again at 80, while equity is now at 40. Hence, the new leverage is 120/40= 3

- Data show that while this is true for households, it is not true for financial firms (Adrian and Shin, 2010 b)

INTRODUCTION (CONT'D)

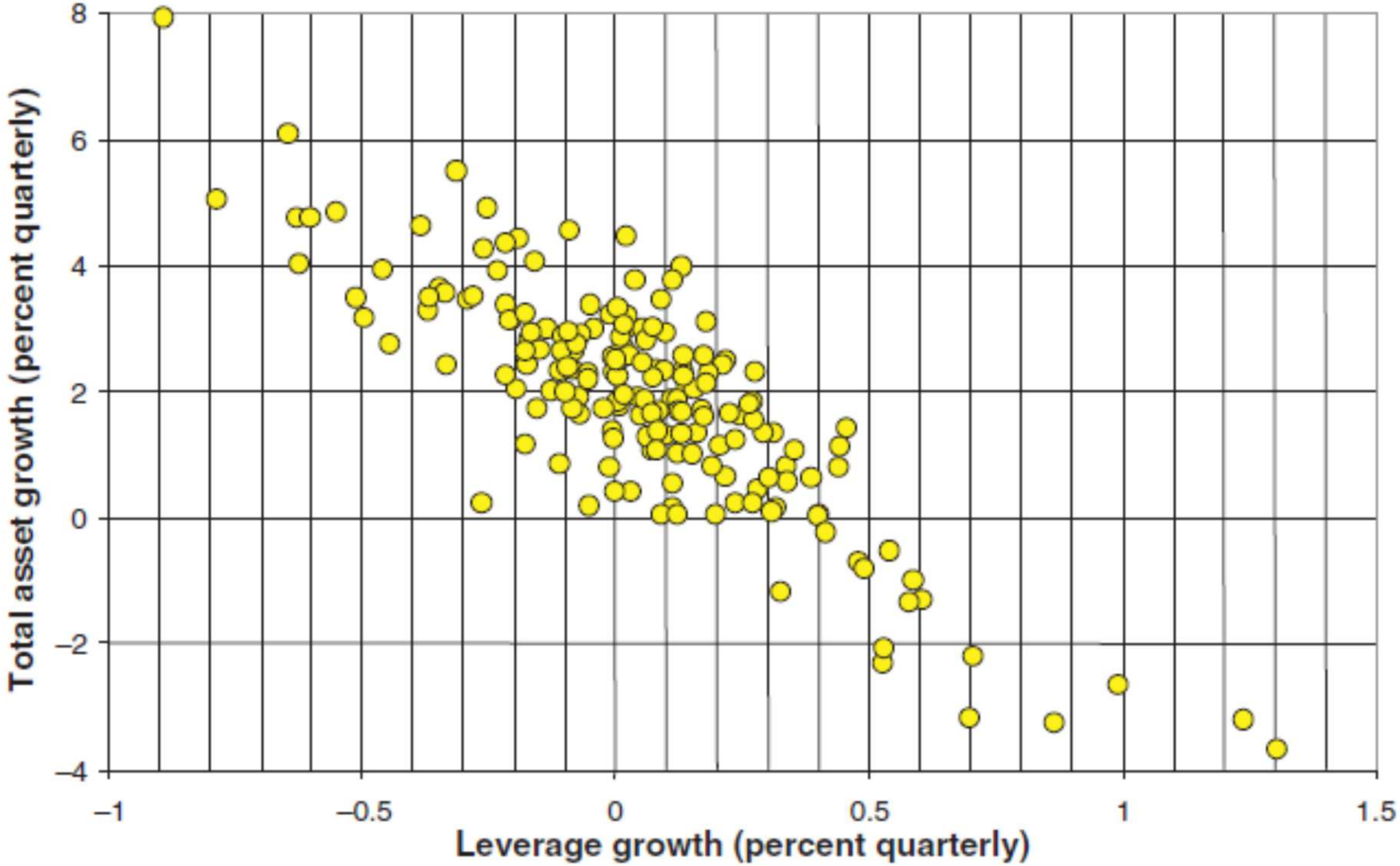


Figure 12

Household sector leverage and total assets. Data taken from the U.S. Flow of Funds, Federal Reserve, 1963–2007.

INTRODUCTION (CONT'D)

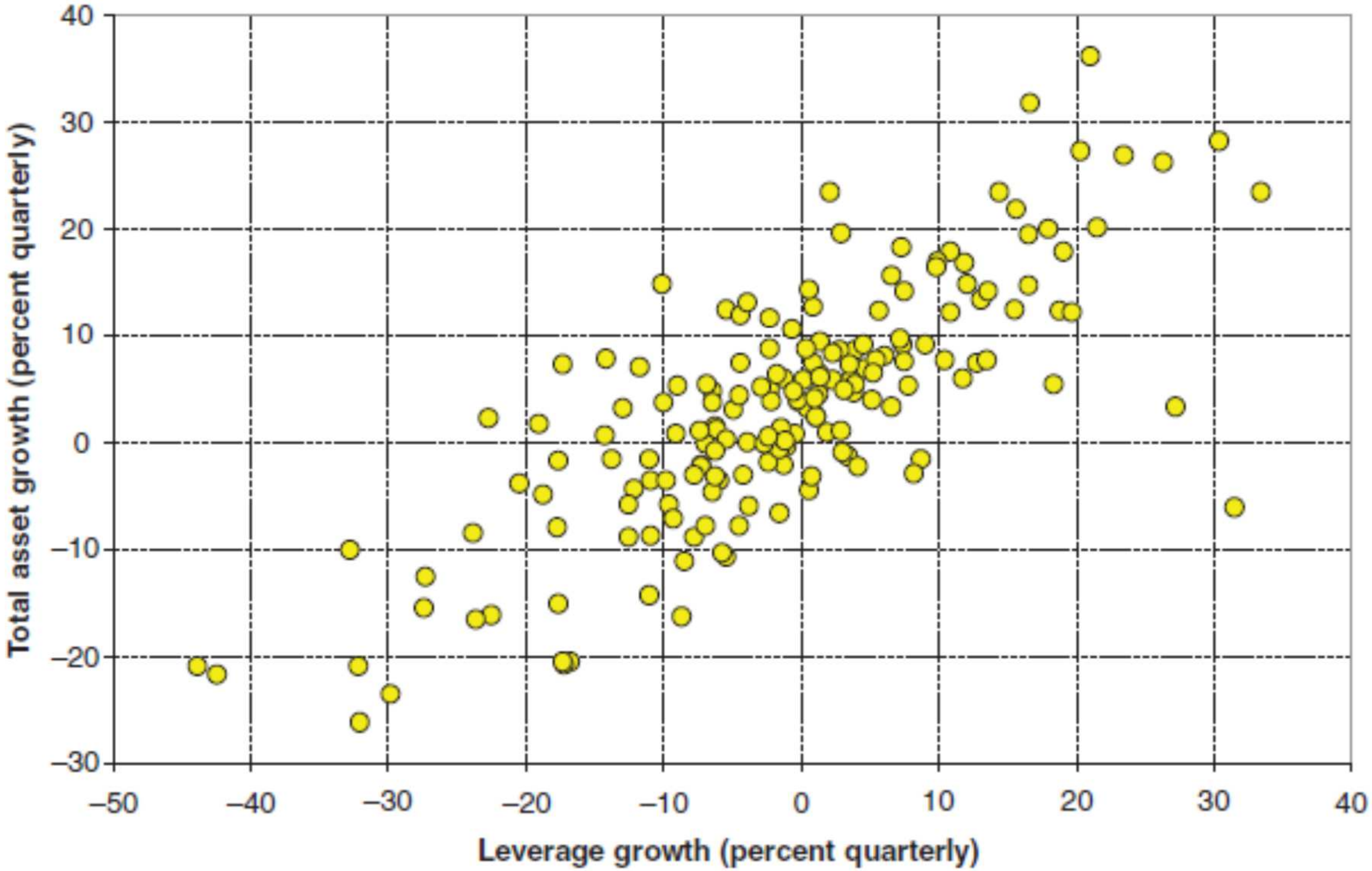


Figure 14

Broker-dealer sector leverage and total assets. Data taken from the U.S. Flow of Funds, Federal Reserve, 1963–2007.

BASICS

- A two-period economy, $(t, t+1)$
- Suppose an investor has to set her asset allocation by choosing between
  1) an amount y_t of a risky security, which costs p_t
  2) cash, c_t (can be negative, if the investor borrows)

- The price of the risky security at time $(t+1)$ is

$$p_{t+1} = (1 + \tilde{r}_{t+1})p_t$$

where $\tilde{r}_{t+1}$ is the outcome of a stochastic process, with

$$\tilde{r}_{t+1} \sim iid\ \Omega(\mu, \sigma^2), \qquad \mu > 0$$

BASICS (CONT'D)

- The investor has an initial capital endowment e_t
- The time t balance sheet is $p_t y_t + c_t = e_t$, and it is consistent with either short or long positions on the risky security

a) Leveraged investor (i.e., a financial intermediary)

Assets		Liabilities	
Securities	$p_t y_t$	Equity	e_t
		Debt	$-c_t$

The leverage is $L_t = \frac{p_t y_t}{e_t}$

BASICS (CONT'D)

b) **Short-only investor** (i.e., a short-only hedge fund)

Assets		Liabilities	
Cash	c_t	Equity	e_t
		Securities	$-p_t y_t$

The leverage is $L_t = \frac{e_t - p_t y_t}{e_t}$

BASICS (CONT'D)

c) **Long-only investor** (i.e., a mutual/pension fund)

Assets		Liabilities	
Cash	c_t	Equity	e_t
Securities	$p_t\, y_t$		

The leverage is $L_t = 1$

Basics (cont'd)

- The evolution of equity (with unchanged asset allocation) is

$$\begin{aligned} e_{t+1} &= p_{t+1}\, y_t + c_t \\ &= p_{t+1}\, y_t + (e_t - p_t\, y_t) \\ &= (p_{t+1} - p_t)\, y_t + e_t \\ &= [(1 + \tilde{r}_{t+1})p_t - p_t]\, y_t + e_t \\ &= \tilde{r}_{t+1} p_t\, y_t + e_t \end{aligned}$$

- The value of the equity in $(t + 1)$ reflects the gains or losses associated to the risky security (since the return on cash is zero)
- Once the accounting has been defined, let us focus on the behavior of the investor

BASICS (CONT'D)

- In what follows we assume the investor is risk-neutral

- Suppose the asset allocation is totally unconstrained. Since the expected return from the risky security is $\mu > 0$ (which correponds to the return from holding cash), it is optimal for the investor to borrow without limits ($-c \to \infty$) and use the debt to buy risky securities
- There is a case for external (prudential regulation) or internal (risk management) mitigation of the risk-taking capacity of a financial intermediary

VALUE-AT-RISK

- A standard approach to limit the risk-taking capacity of a risk-neutral financial intermediary is to impose a Value-at-Risk (VaR) constraint
- The idea consists in maximizing the return on the invested capital, forcing the risk of the portfolio to remain below an «acceptable» threshold
- The «acceptable» risk threshold is given by the VaR associated to the probability to become insolvent during the next period

VALUE-AT-RISK (CONT'D)

- Let α be the VaR confidence level, so that (1 - α) represents the maximum probability to go bankrupt the following period
- Bankruptcy occurs whenever $e_{t+1} \leq 0$, occurring if

$$\tilde{r}_{t+1} p_t y_t + e_t \leq 0$$

that is

$$\tilde{r}_{t+1} \leq -\frac{e_t}{p_t y_t}$$

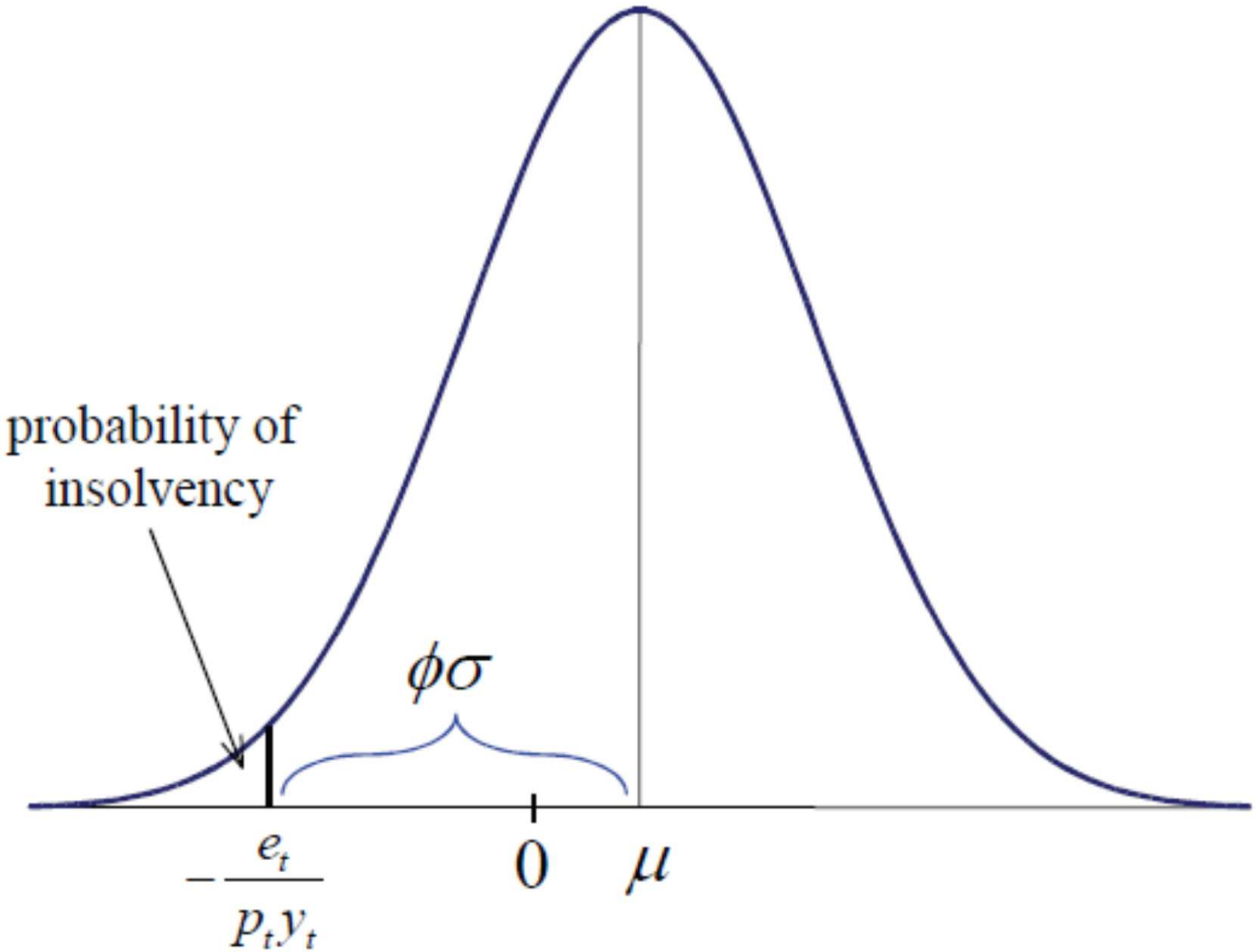
VALUE-AT-RISK

- Define ϕ as constant such that

$$Prob(\tilde{r}_{t+1} \leq \mu - \phi\sigma) = 1 - \alpha$$

- Hence, $\phi\sigma$ is the VaR associated to the stochastic return $\tilde{r}_{t+1}$ at the confidence level α, when the expected return is μ
- The problem is now simply that of choosing an asset allocation with an amount of risky asset y such that the probability to become insolvent at time $(t + 1)$ is not higher that $(1 - \alpha)$

VALUE-AT-RISK (CONT'D)



VALUE-AT-RISK (CONT'D)

- The probability of insolvency is exactly equal to (1 - α) when

$$-\frac{e_t}{p_t y_t} = \mu - \phi\sigma$$

that is

$$\mu + \frac{e_t}{p_t y_t} = \phi\sigma$$

- Given the initial position in equity (e_t), the expected return on the risky asset (μ), its riskiness (σ) and the acceptable threshold for risk (ϕ), this condition identifies the total amount of risky asset to be held in the portfolio

VALUE-AT-RISK (CONT'D)

$$p_t y_t = \frac{e_t}{\phi\sigma - \mu}$$

- Does an investor want to hold less?
- Since $\mu > 0$, the expected value of time $(t+1)$ equity is

$$E(e_{t+1}) = \mu p_t y_t + e_t$$

which is increasing in $p_t y_t$. Hence the answer is no!

- The optimal allocation in the risky security is

$$y_t{}^{*} = \frac{\left(\frac{e_t}{\phi\sigma - \mu}\right)}{p_t}$$

VALUE-AT-RISK (CONT'D)

- This has an interesting implication for leverage
- Notice that the leverage is equal to

$$L = \frac{p_t y_t}{e_t} = \frac{e_t}{\phi\sigma - \mu}\frac{1}{e_t} = \frac{1}{\phi\sigma - \mu}$$

which is constant, with a dependence on parameters

$$L = L\,(\underset{-}{\phi}\,,\,\underset{-}{\sigma}\,,\,\underset{+}{\mu}\,)$$

VALUE-AT-RISK (CONT'D)

If the price of the risky asset varies, an investor using a VaR approach in managing her asset allocation wants to keep the leverage constant («leverage targeting»)

As a result, the demand curve for a risky asset has a positive slope (… demand more asset when its price increases), while the supply curve has a negative slope (… sell assets if the price decreases)

This is precondition for the amplification of shocks

UPWARD-SLOPING DEMAND

- The rate of change of equity is

$$\frac{e_{t+1} - e_t}{e_t} = \tilde{r}_{t+1} \frac{p_t y_t}{e_t} = \tilde{r}_{t+1} L$$

- While the proportional variation of the position (i.e., nominal value) in the risky asset as its price varies (keeping the structure of the asset allocation constant) is

$$\frac{p_{t+1} y_t - p_t y_t}{p_t y_t} = \frac{(1 + \tilde{r}_{t+1}) p_t y_t - p_t y_t}{p_t y_t} = \tilde{r}_{t+1}$$

For a leveraged investor, equity increases L times the increase in the value of assets

UPWARD-SLOPING DEMAND (CONT'D)

- As the price increases, the leveraged investor has a much higher equity
- Remember that under the VaR risk-management approach it is optimal to keep L constant
- How does she ricompose her portfolio after the price has changed?
- Keeping L constant between t and $(t+1)$ means

$$\frac{p_t y_t}{e_t} = \frac{p_{t+1} y_{t+1}}{e_{t+1}} = L$$

- Which can be rewritten to obtain

$$\frac{y_{t+1}}{y_t} = \frac{e_{t+1}/e_t}{p_{t+1}/p_t} = \frac{1 + \tilde{r}_{t+1} L}{1 + \tilde{r}_{t+1}}$$

UPWARD-SLOPING DEMAND (CONT'D)

- Finally, the proportional variation of the asset's holding is

$$\frac{\Delta y_{t+1}}{y_t} = \frac{y_{t+1} - y_t}{y_t} = \frac{\tilde{r}_{t+1}}{1 + \tilde{r}_{t+1}}(L - 1)$$

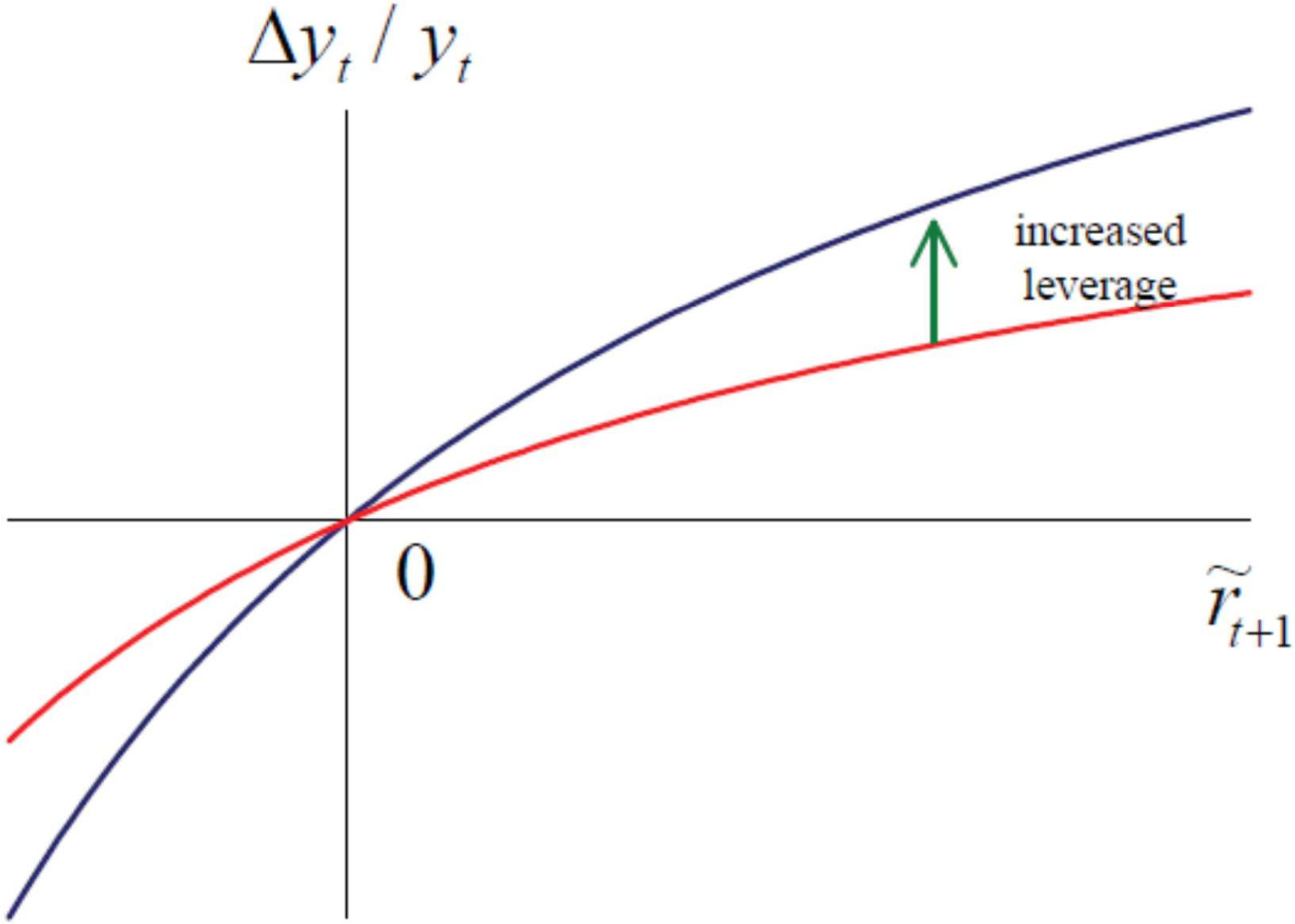
- We can assess how the demand for the risky asset responds to a variation of the return $\tilde{r}_{t+1}$

$$\frac{d[(L-1)r(1+r)^{-1}]}{dr} = \frac{1}{(1+r)^2}(L-1) > 0$$

A higher price causes a higher demand for the asset

The higher the leverage L, the steeper the demand response to price changes

UPWARD-SLOPING DEMAND (CONT'D)



ADRIAN-SHIN

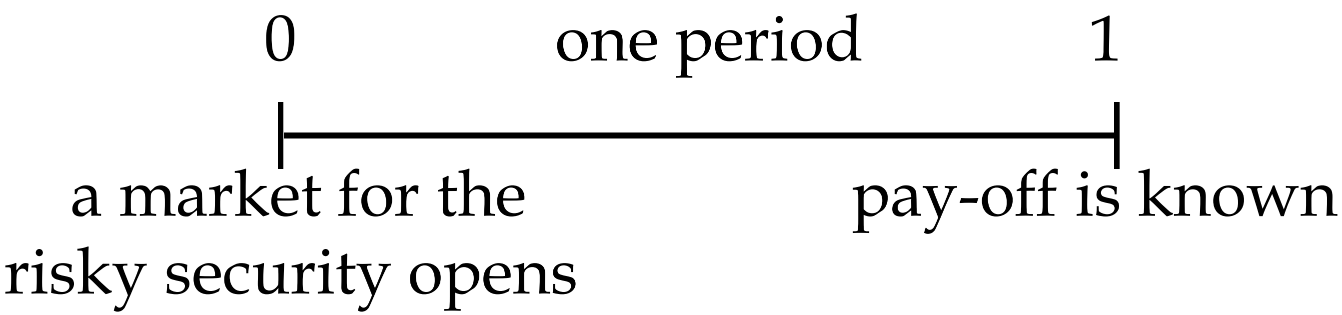
- Adrian-Shin (2010a):
  - Variations in asset prices affect the risk-taking capacity of the financial intermediary sector
  - The mechanism operates through an expansion/compression of balance sheets
  - Pro-cyclical fluctuations in leverage create an externality which amplifies shocks

The Intuition

- An investor, by trying to minimize her own risk, increases the risk for all the others
- This externality generates systemic risk
- At odds with models in which systemic risk is due to a «domino» effect, the transmission mechanism of the externality operates through the portfolio allocation decisions of market participants, and their influence on the pricing of risk
- In this model
  - Debt is risk-free
  - Intermediaries are not linked by mutual debt/credit obligations

THE MODEL SET-UP

- A market for a risky security (RS) operates according to the following timing structure



- At time t = 0, the t = 1 pay-off of the RS is defined by a uniformly distributed random variable

$$\widetilde{\omega} \sim [q - z, q + z], \qquad with\, z > 0$$

$$E(\widetilde{w}) = q > 0; \quad {\sigma_\omega}^2 = \frac{z^2}{3}$$

THE MODEL SET-UP (CONT'D)

- Furthermore, there exists a risk-free security with a null net return, say cash (c)
- Notation
  - p = price of the RS
  - y = number of units of the RS held in the portfolio
  - e = initial endowment of capital
  - W = wealth in $t = 1$
    $= (\tilde{w} - p)y + e$
- Thus, final wealth is a random variable. It can be rewritten as

$$W = \tilde{w}y + (e - py) = \tilde{w}y + c$$

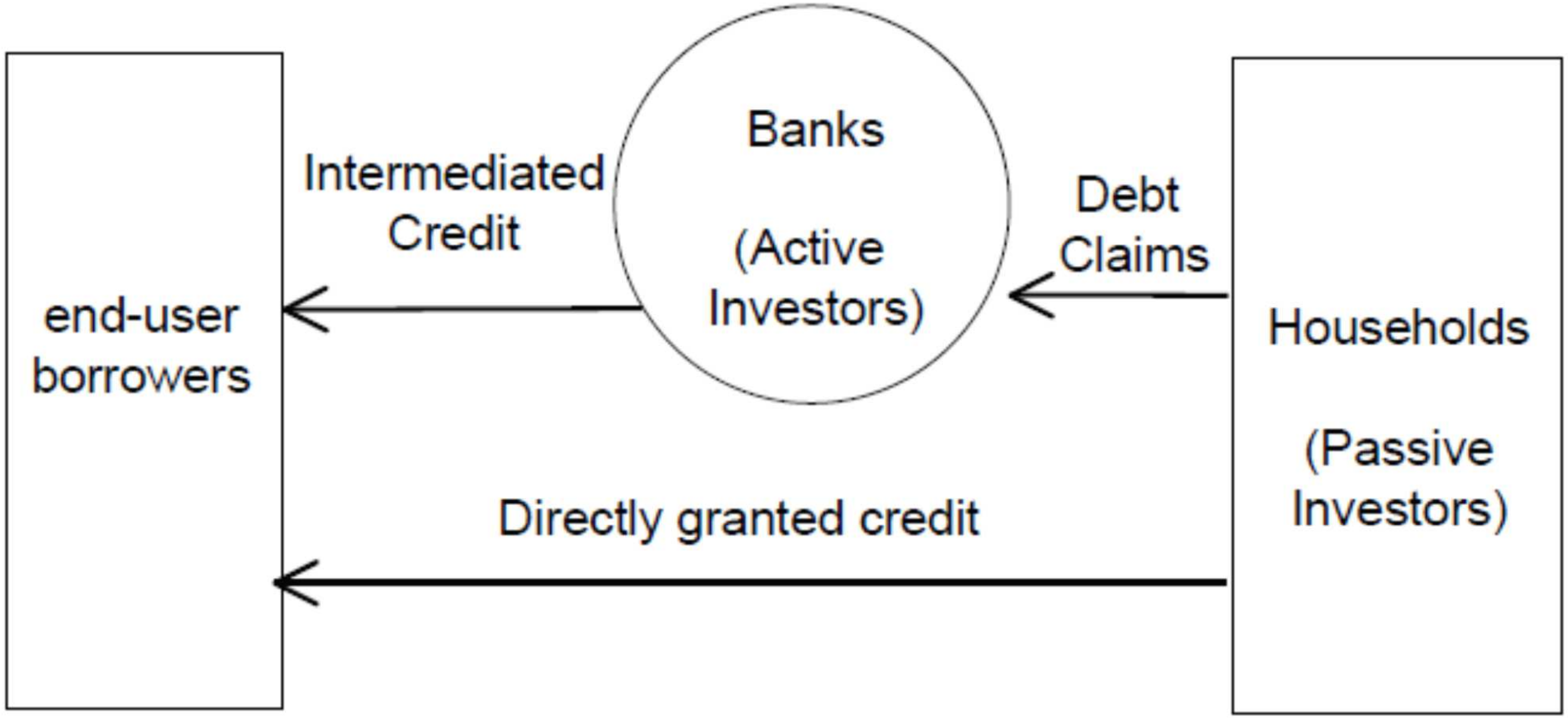
where $Var(W) = y^2\sigma^2 = \frac{z^2y^2}{3}$, while c could be positive or negative

THE MODEL SET-UP (CONT'D)

- Let us introduce two classes of investors
  1) passive: not leveraged (ex., households)
  2) active: leveraged (ex., banks)

- We assume that active investors get funds by issuing risk-free debt (where the interest rate is normalized to zero, i.e. cash) bought by passive investors, and use these funds to buy risky securities issued by end-user borrowers (say, firms)
- Passive investors, in turn, can hold both risk-free debt issued by banks and risky securities issued by end-user borrowers

THE MODEL SET-UP (CONT'D)



i. Passive investors hold risk-free debt and risky securities
ii. Active investors hold risky securities, and fund their holdings by internal capital (e) and risk-free debt (c)

PASSIVE INVESTORS

- Passive investors choose their asset allocation by maximizing a mean-variance utility function defined on the final value of the portfolio

$$\max U = E(W) - \frac{1}{2\tau}\sigma_W{}^2$$

where τ is a parameter measuring the risk tolerance

- Substituting

$$max\, U(y) = [qy + (e - py)] - \frac{1}{2\tau}\frac{z^2y^2}{3}$$

Passive investors (cont'd)

- The first-order condition w.r.t. y is

$$q - p - \frac{2}{6\tau} y z^2 = 0$$

i.e.

$$y = (q - p)\frac{3\tau}{z^2}$$

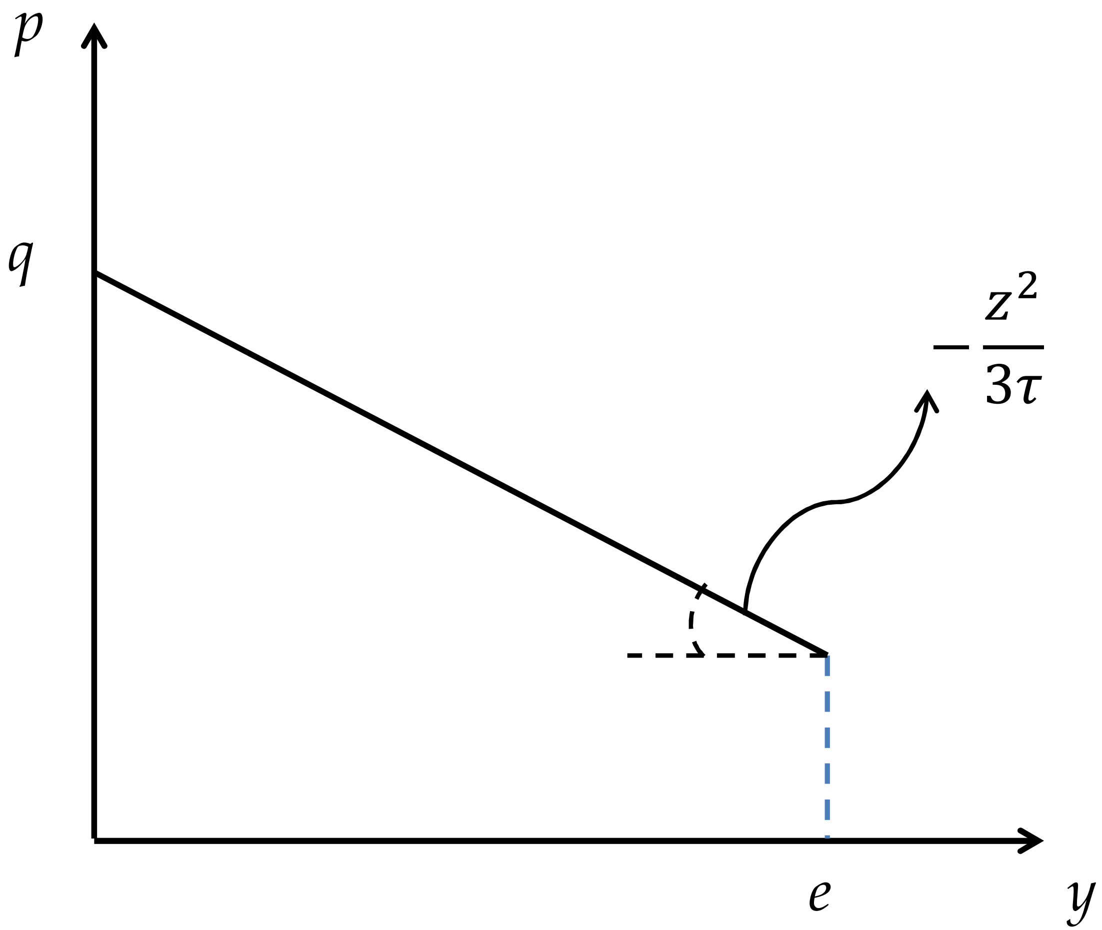
- The demand for risky securities from a passive investor is

$$y = \begin{cases} \frac{3\tau}{z^2}(q-p) & if \quad q > p \\ 0 & if \quad q < p \end{cases}$$

Demand occurs only if the price is below the expected pay-off

- Under mild aggregation conditions, we can assume it represents the aggregate demand of the passive sector as a whole

PASSIVE INVESTORS (CONT'D)



a. If $z \uparrow \;\rightarrow$ steeper demand curve

b. If $\tau \uparrow \;\rightarrow$ flatter demand curve

ACTIVE INVESTORS

- Active investors are leveraged. They are risk-neutral, and fund themselves by issuing risk-free debt under a VaR constraint
- Operationally, this means they must hold a buffer of equity big enough to assure that their probability of default is below a given threshold
- The objective function is linear in the portfolio's expected pay-off, so that their problem is

$$\max_{y} E(W)$$

$$s.t. \qquad VaR \leq e$$

ACTIVE INVESTORS (CONT'D)

- This problem admits two solutions, depending on

  1) $p > q \rightarrow$ no risky securities (and no debt for financing)
  2) $p < q \rightarrow E(W)$ is increasing in y, corner solution

- The solution 2) implies that the portfolio contains only risky securities, with the investor borrowing up to the limit allowed by the VaR constraint, which becomes binding, $VaR = e$
- Recall an active investor issues risk-free debt with a null return

$$c = py - e$$

ACTIVE INVESTORS (CONT'D)

- In order for this debt to be risk-free (recall that funds are invested by the bank in a risky asset), the equity buffer must be such that

$$py - e = (q - z)y$$

i.e.

$$[p - (q - z)]y \leq e$$

where the lhs term represents the max possible loss (VaR) with respect to the current market value of the investment in risky asset

- An active investor faces a constraint on the maximum amount of debt she can issue, and the constraint tightens as market prices fall

ACTIVE INVESTORS (CONT'D)

- Recall the proble admits a corner solution, so that the constraint binds

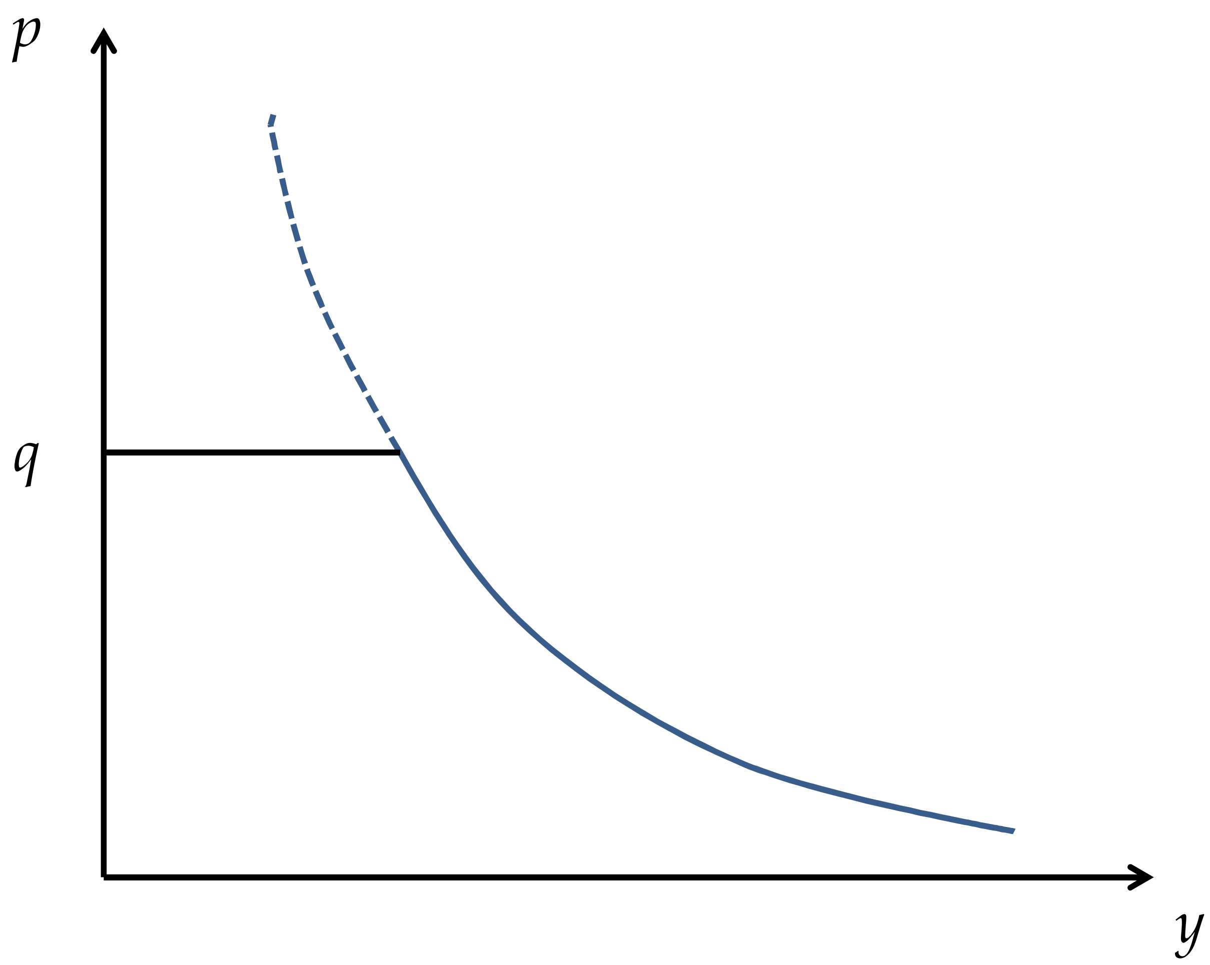
$$py - (q - z)y = e$$

- The demand for the risky asset from an active investor is

$$y_A = \frac{e}{z - (q - p)}$$

- This demand function is parameterized by the investor's available equity. Since it is linear in e, we can interpret it as the market demand function of the active segment

ACTIVE INVESTORS (CONT'D)



Market Clearing

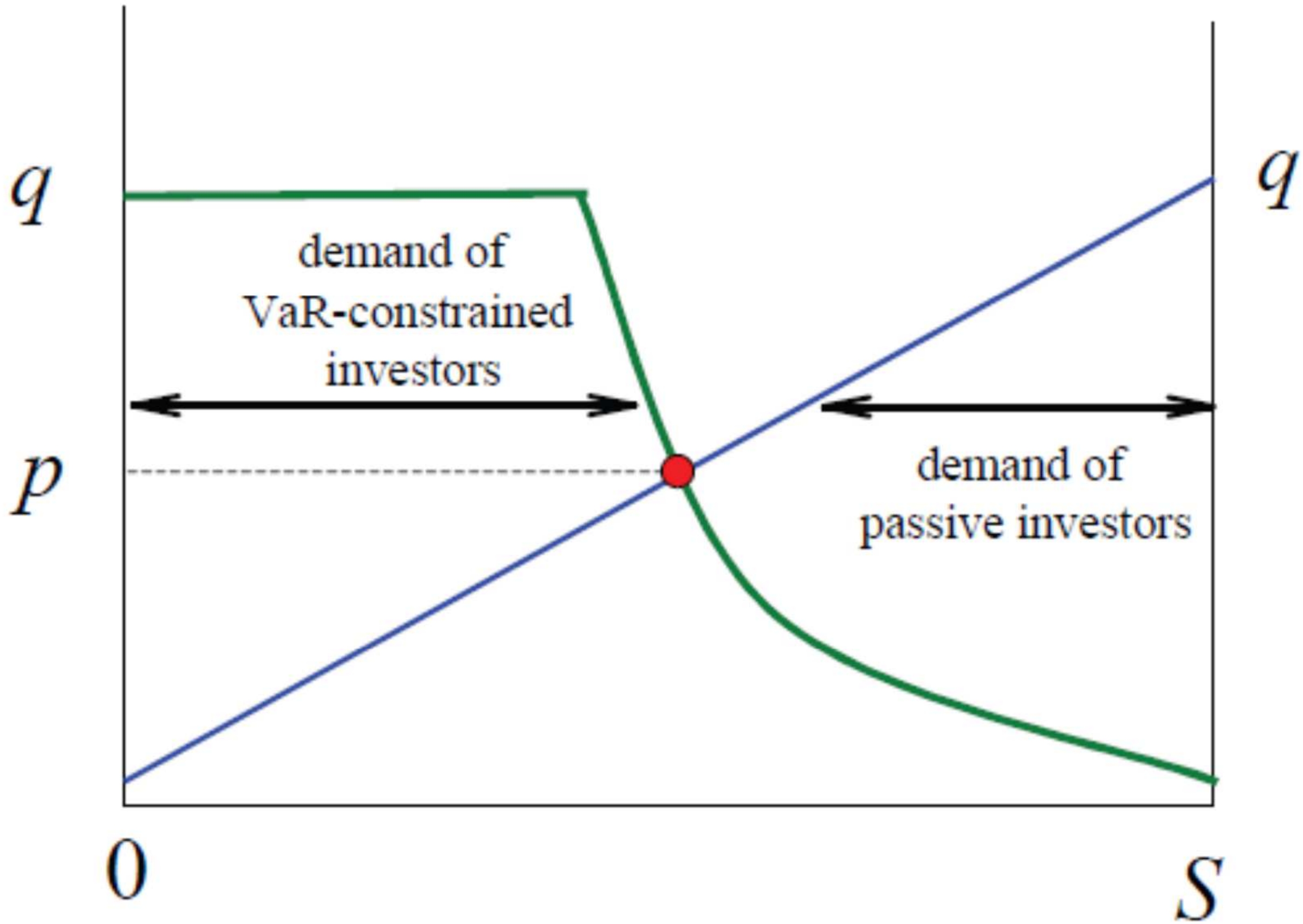
- Let S be the (exogenously given) fixed supply of risky asset
- Market clearing implies

$$y_A + y_B = S$$

- For expositional convenience, think about it as

$$y_A = S - y_B$$

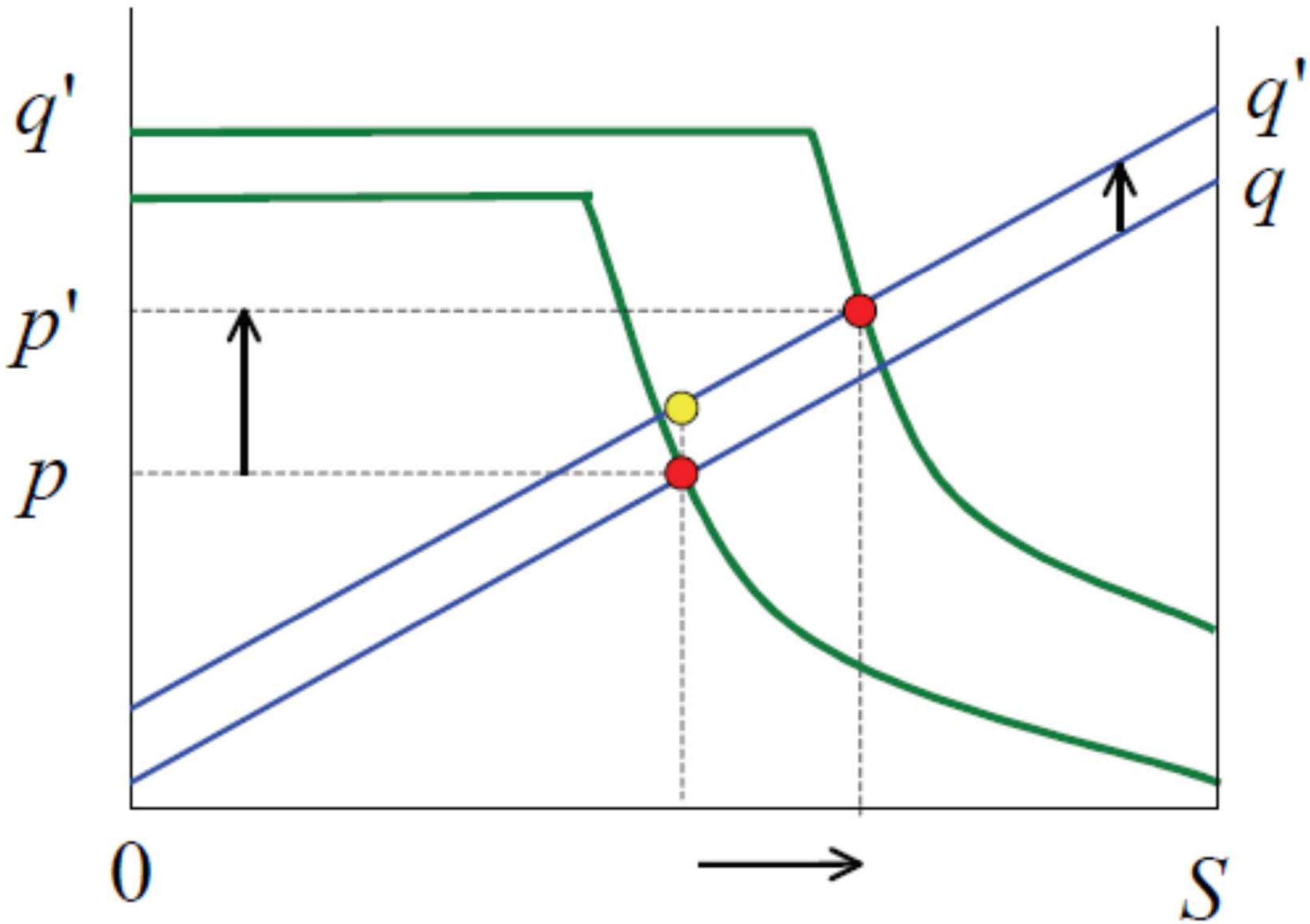
Market Clearing (cont'd)



AMPLIFICATION

- What does it happen if the fundamentals of the risky security improve?
- Suppose $q' > q$, due for instance to an improved macroeconomic outlook, driving to a lower default probability for borrowers
- A higher q makes both demand functions shift upwards
- The adjustment in the balance sheet structure of (leveraged) active investors causes an amplified response
- The total market demand for the risky asset increases with its market price $\rightarrow$ upward-sloping market demand function

AMPLIFICATION (CONT'D)



THE MECHANICS

- Let's go deep on the dynamics
- Let e' be the new equity level of the leveraged investors resulting from the higher price p', but before any adjustment in their balance sheet
- Recall the intial amount of debt is $(q - z)y$
- Hence

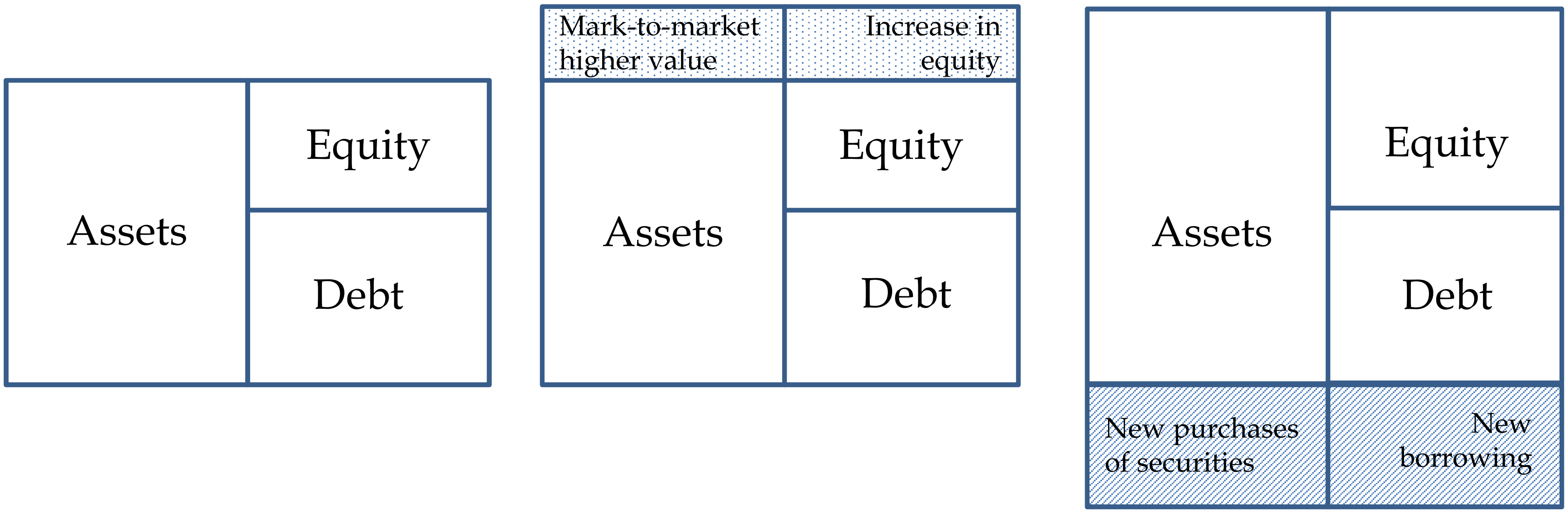
$$e' = p'y - (q - z)y = (z + p' - q)y > e$$

THE MECHANICS (CONT'D)

Assets		Liabilities	
Securities	py	Equity	e
		Debt	$py - e$

⇒ Due to the VaR constraint, this is $(q - z)y$

i) Initial balance-sheet   *ii*) After the shock, $q' > q$ *iii*) After the adjustment



THE MECHANICS (CONT'D)

- Combining the new equity level *e*′ (increased due to the higher market value of assets) with an unchanged level of debt means that the leverage is decreased.
- The investor has some «spare» equity, that can be used to expand the balance sheet until the leverage returns to its target

Leveraged investors respond to an increase in the mark-to-market value of their assets (due to a higher market price) by demanding more

THE MECHANICS (CONT'D)

Mark-to-market effect

$$e' = (z + p' - q)y$$
$$= p'y - (q - z)y$$

Ricomposition of the b/s

$$e' = (z + p' - q')y'$$
$$= p'y' - (q' - z)y'$$

$$(z + p' - q)y = (z + p' - q')y'$$

i.e.

$$y' = \left(\frac{z+p'-q}{z+p'-q'}\right) y = \left(\frac{z+p'-q+q'-q'}{z+p'-q'}\right) y$$
$$= \left(1 + \frac{q'-q}{z+p'-q'}\right) y$$

THE MECHANICS (CONT'D)

- From the market clearing condition

$$y_A + y_P = S \quad \rightarrow \quad y_P = (S - y_A)$$

- and the demand function of passive investors

$$y_P = \frac{3\tau}{z^2}(q - p)$$

- we obtain

$$p' - q' = \frac{z^2}{3\tau}(y' - S)$$

THE MECHANICS (CONT'D)

- Substitute in the new demand of leveraged investors to obtain

$$y' = \left(1 + \frac{q' - q}{z + \frac{z^2}{3\tau}(y' - S)}\right) y$$

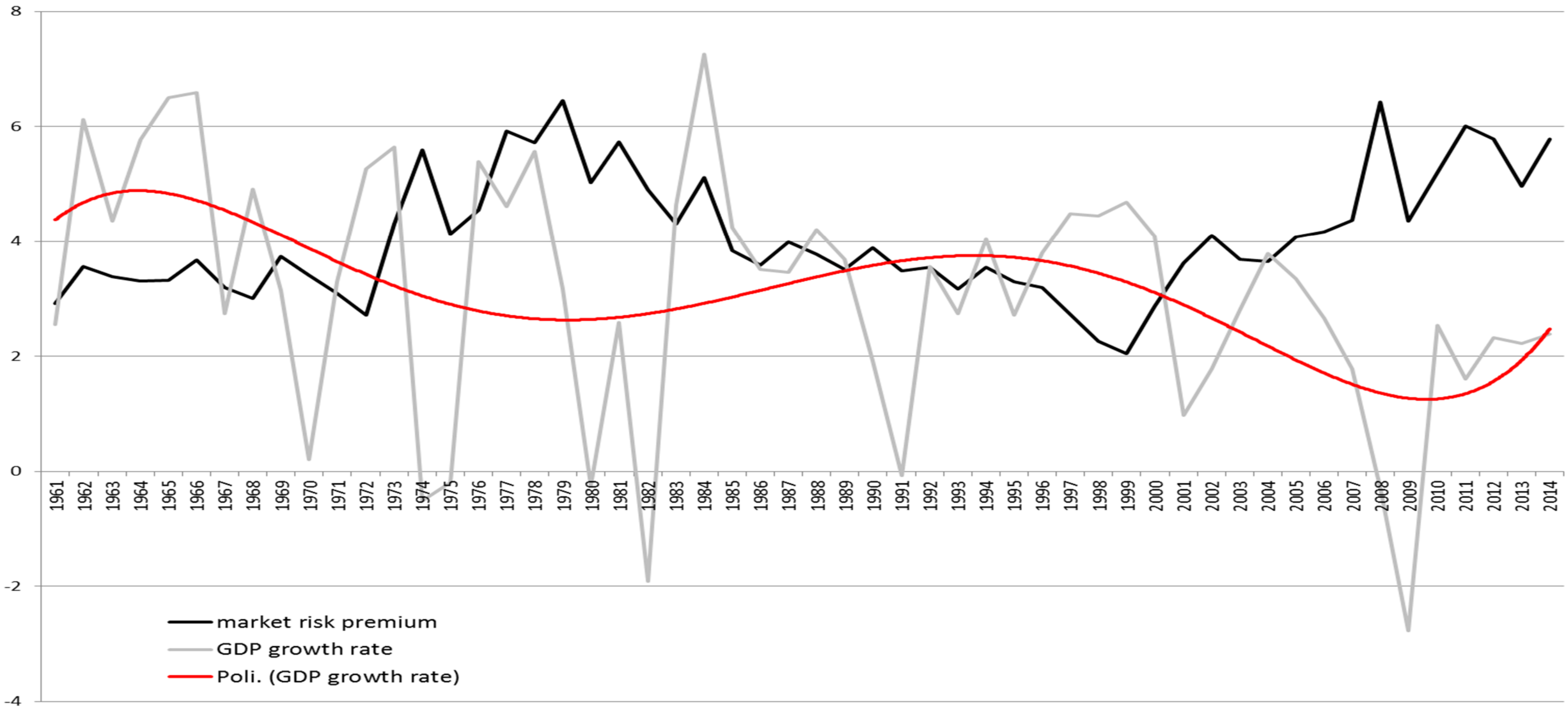
A shock to fundamentals ($q' > q$) causes a response of identical sign in the demand from active investors ($y' - y > 0$), that amplifies the shock

THE MECHANICS (CONT'D)

- The higher the leverage, the stronger the amplification, since ($y' - y$) depends negatively on z (fundamental risk), and when z is low the leverage increases
- The spread $q - p$ represents a measure of the risk premium
- Since $q - p = \frac{z^2}{3\tau}(S - y_A)$, we find that the risk premium is lower whent the size of the leveraged sector w.r.t. the total market is higher

Fluctuations in Market Risk Premia

- A large empirical literature suggests that risk premia on stocks or corporate bonds are large and *countercyclical*.



Correlation coefficient ρ = -0.295

MARKET RISK PREMIA (CONT'D)

- Total supply S is fixed
- Market clearing, $y_A + y_B = S$, implies

$$\frac{e}{z-(q-p)} + \frac{3\tau}{z^2}(q-p) = S$$

- Impose a restriction: the demand for risky assets by active investors is strictly positive, $y_A = S - y_B > 0$, that is

$$\frac{3\tau}{z^2}(q-p) < S$$

PROPOSITION

The expected return from the risky security is strictly decreasing in q

MARKET RISK PREMIA (CONT'D)

- PROOF
  - Redefine the expected return as $\pi = 1 - \frac{p}{q}$, and notice that $\pi \in (0,1)$
  - $\pi = 0$ if $p = q$, i.e. the premium required to hold a risky asset over a safe asset (cash) is null when the actual price is equal to its expected value
  - Substituting in the market clearing condition one gets

$$e + \frac{3\tau}{z^2} q\pi(z - q\pi) = S(z - q\pi)$$

define the implicit function $F(\pi, q) = 0$,

$$F(\pi, q) = e + \frac{3\tau}{z^2} q\pi(z - q\pi) - S(z - q\pi) = 0$$

Market Risk Premia (cont'd)

- (proof cont'd)
  - By the implicit function theorem

$$\frac{d\pi}{dq} = -\left[\frac{\partial F}{\partial q} \Big/ \frac{\partial F}{\partial \pi}\right]$$

  - The proposition is true iff $\frac{d\pi}{dq} < 0$
  - The expected return on the risky asset decreases as the fundamentals on the economy improve. Stated differently, the risk premium decreases when the economy is booming

Market Risk Premia (cont'd)

- (proof cont'd)
  - It can be shown that the imposed restriction on the positivity of the demand for risky asset by leveraged investors implies

$$\frac{\partial F}{\partial \pi} > 0; \quad \frac{\partial F}{\partial q} > 0$$

which gets

$$\frac{d\pi}{dq} < 0$$

Market Risk Premia (cont'd)

- When the fundamentals on the economy improve, leveraged investors register an increase in the mark-to-market valuation of their assets.
- The greater net worth creates a «spare» capacity for new indebtedness, and an expansion of the balance sheet.
- The demand for risky asset increases, along with the market price, and the risk premium shrinks

A General Feature

- The whole story depends on a general feature (here at work due to the VaR/risk management constraint)
- A sufficient pre-condition for a financial crises is some balance sheet mechanism according to which ***MARKET CONDITIONS AND FINANCING CONDITIONS WORSEN AT THE SAME TIME***
- Krishnamurthy (2010) discusses how the same occurs in models with
  - Margin requirements and margin calls
  - Collateral constraints
  - Optimal «skin in the game» contracts between outside investors and inside managers

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- Krishnamurthy, A. (2010), Amplification mechanisms in liquidity crises, *American Economic Journal: Macroeconomics*, **2**:1-30.